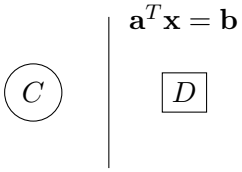
Separation theorem

How to show that two convex sets are disjoint?

Theorem 1. *If $C, D \subseteq \mathbb{R}^d$ are convex sets and $C \cap D = \emptyset$, then there exists a hyperplane separating C and D. That is, there exists $\mathbf{a} \in \mathbb{R}^d$, $b \in \mathbb{R}$ such that*
$\forall \mathbf{x} \in C, \mathbf{a}^T\mathbf{x} \leq b$
$\forall \mathbf{x} \in D, \mathbf{a}^T\mathbf{x} \geq b$

Separation can be strict if C and D closed and one bounded.



1: Why is the theorem true if C and D are compact? (in $\mathbb{R}^d$ compact means closed and bounded)

2: Why is the theorem true if C compact and D closed?

3: Why is the theorem true in general?